Light paths in a ferronematic cell

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The control of the light trajectories in a liquid crystal is studied through a theoretical model based in the variation of the molecular orientation of a ferronematic material. In this model, the director field is mapped into a Riemannian space where the light paths are obtained numerically through the calculation of the geodesics for the effective geometry perceived by light.

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I. INTRODUCTION

The liquid crystals [1] are used in large scale by the current industry due to their potential for technological applications. As a large number of applications come from the optical response of these materials [2], the study of the light propagation in liquid-crystalline media has great relevance. For the light, the liquid crystals constitute an inhomogeneous space (birefringence) [3] that in general has orientation defined by boundary conditions to which the liquid crystal is subjected. We previously developed in [4] a theoretical model to describe the light propagation in nematics with molecular orientation imposed by the presence of topological defects [5]. In this model [4], the light rays are interpreted as being geodesics in the Riemannian space [6] into which the optical medium composed by the nematic is mapped. The trajectories [7] are obtained from a metric (line element) associated with the effective geometry perceived by light. As a result, we have a lensing effect whose control can be determined simply by a topological parameter of the system.

Another system that can be modeled in a similar way is what results from the introduction of a magnetic material (ferroparticles) in the composition of a nematic liquid crystal [8]. From this mixture, we have the ferronematics [9], whose ability to align under the action of magnetic fields [10] allows the manipulation of its molecular configuration. In general, the physical properties of liquid crystals depend on the molecular orientation displayed at each phase [11]. Therefore, the variation of parameters such as temperature or magnetic field acts as a way to change these properties through the induction of orientational order.

In this paper, we perform a study of the light trajectories in a ferronematic cell using the model proposed in [4]. The light trajectories are calculated numerically and the control parameter that characterizes the changes in the effective geometry perceived by light is given by the magnetic field $\vec{H}$ applied.

II. THEORETICAL MODEL

The purpose of the model described in this section is to give us a geometric description for the light trajectories in a liquid crystal. The main idea of model is in the comparison of Fermat's principle to the variational principle that determines the geodesics in the Riemannian geometry. From Fermat's principle, we have that the length of the optical path traveled by a light beam can be calculated by

$$\mathcal{F} = \int_A^B N d\ell, \tag{1}$$

where $d\ell$ is the element of arc length along the path between points A and B. Then, among all possible paths between the generic points A and B, Fermat's principle for the extraordinary rays grants us that the path actually followed by the energy is the one that minimizes $\mathcal{F}$. In Eq. (1), the effective refractive index N felt by the light [3] is given by a function that depends on the local position and local direction of the light beam

$$N^2 = n_o^2 \cos^2 \beta + n_e^2 \sin^2 \beta, \tag{2}$$

where $\beta = (\widehat{\vec{n}, \vec{S}})$ is the local angle between the director $\vec{n}$ and the Poynting vector $\vec{S}$ (see Fig. 1 of [4]), n_o is the ordinary refractive index, and n_e is the extraordinary refractive index of the molecule of liquid crystal. The refraction angle β is given by the scalar product between the director $\vec{n}$ and the local propagation direction of the beam $\vec{T}$ tangent to the trajectory, therefore

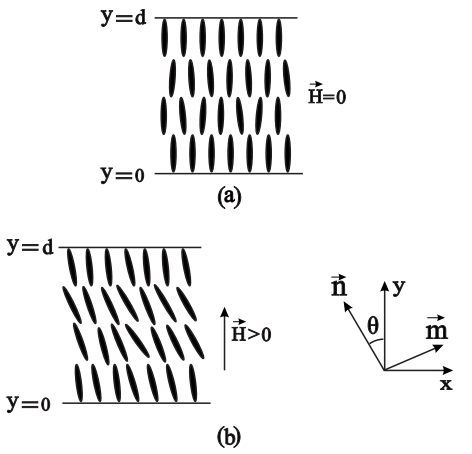

FIG. 1. Molecular orientation θ of ferronematic cell under the influence of the magnetic field applied $\vec{H}$.

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$$\cos \beta = \vec{n} \cdot \vec{T}, \tag{3}$$

where $\vec{T}$ is unitary and is in the direction of $\vec{S}$ in each point.

In Riemannian geometry, the line element ds depends on the position coordinates x^i of the point of the manifold under consideration, that is,

$$ds^2 = \sum_{i,j} g_{ij} dx^i dx^j, \tag{4}$$

where $g_{ij}=g_{ij}(x^i)$ is the metric tensor. The geodesic joining points A and B in such manifold are obtained by minimizing $\int ds$, just like Fermat's principle. This leads to a nice interpretation of the light paths as geodesics in an effective geometry. Thus, we may identify

$$N^2 d\ell^2 = \sum_{i,j} g_{ij} dx^i dx^j. \tag{5}$$

The meaning of this equation is the following: the line element of the optical path, in a Euclidean space with refractive properties, is identified with the line element of an effective geometry characterized by g_{ij}.

As n_o and n_e are functions of T, the temperature is included [12] as a parameter of characterization of the geometry [13] effective by

$$n_o = \langle n \rangle - \frac{1}{3}\Delta n, \tag{6}$$

$$n_e = \langle n \rangle + \frac{2}{3}\Delta n, \tag{7}$$

given in terms of the birefringence Δn and of its average value $\langle n \rangle$. In Eqs. (6) and (7), the behavior of $\langle n \rangle$ as function of the temperature [12] is given experimentally through a linear dependence given by

$$\langle n \rangle = A - BT, \tag{8}$$

where the parameters A and B are obtained experimentally. The birefringence can be written in terms of the approximated order parameter [14] $S=(1-\frac{T}{T_c})^{\alpha}$ as

$$\Delta n = (\Delta n)_0 \left(1 - \frac{T}{T_c}\right)^{\alpha}, \tag{9}$$

where $(\Delta n)_0$ is the birefringence in the crystalline state, α is a constant associated to the material, and T_c is the isotropic-nematic transition temperature. Therefore, substituting the Eqs. (8) and (9) into Eqs. (6) and (7), we have

$$n_o = A - BT - \frac{(\Delta n)_0}{3}\left(1 - \frac{T}{T_c}\right)^{\alpha} \tag{10}$$

and

$$n_e = A - BT + \frac{2(\Delta n)_0}{3}\left(1 - \frac{T}{T_c}\right)^{\alpha}. \tag{11}$$

Through the expressions (10) and (11) for the refractive indices of the molecule and the expression for the refractive angle β in a specific coordinates system, we can calculate the effective refractive index N and the coefficients of the metric tensor g_{ij} using the Eq. (5). With g_{ij}, we have the Riemannian line element given by Eq. (4) from which we calculate the geodesics equations [6] with

$$\frac{d^2 x^i}{dt^2} + \sum_{j,k} \Gamma^i_{jk} \frac{dx^j}{dt}\frac{dx^k}{dt} = 0, \tag{12}$$

where t is a parameter along the geodesic and Γ^i_{jk} are the Christoffel symbols given by

$$\Gamma^i_{jk} = \frac{1}{2}\sum_m g^{mi}\left\{\frac{\partial g_{km}}{\partial x^j} + \frac{\partial g_{mj}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m}\right\}. \tag{13}$$

III. EFFECTIVE GEOMETRY

We can use the geometric model described previously to find the light paths in a ferronematic material. The optical medium considered consists of a ferronematic cell with thickness $d \approx 250$ μm from a mixture of magnetic particles added to the nematic liquid crystal 5CB or 4-cyano-4-n-pentylbiphenyl [15–17]. The molecular structure of the cell is given by the geometry (in the equilibrium) of the ferronematic studied in [18], whose orientation under the influence of the external magnetic field $\vec{H}$ is given by [19]

$$\theta(y) = \theta = \frac{M_s f_0 H(d-y)y}{8k_{33}}, \tag{14}$$

considering the limit of small distortions of the director $\vec{n}$ related to the initial position where $\vec{H}=0$ (see Fig. 1). In the solution presented by the Eq. (14), f_o is the volume concentration of ferroparticles in nematic, d is the thickness of the sample (ferronematic layer), M_s is the saturation magnetization of ferroparticles, and k_{33} is the Frank elastic constant.

The components of the director are given by

$$n_x = -\sin \theta,$$

$$n_y = \cos \theta, \tag{15}$$

and the refractive angle β is calculated by the Eq. (3) where $\vec{T}$ is given by $\vec{T}=(\dot{x},\dot{y})$ [20]. Substituting the expression found for β in Eq. (2), we have the effective refractive index N. With that, we calculate the coefficients of the metric tensor g_{ij} through Eq. (5) so that

$$g_{11} = n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta,$$

$$g_{12} = g_{21} = \frac{(n_e^2 - n_o^2)}{2}\sin 2\theta,$$

$$g_{22} = n_o^2 \cos^2 \theta + n_e^2 \sin^2 \theta. \tag{16}$$

Thus, the effective metric is given by

$$ds^2 = (n_o^2 \sin^2 \theta + n_e^2 \cos^2 \theta)dx^2 + (n_o^2 \cos^2 \theta + n_e^2 \sin^2 \theta)dy^2 + (n_e^2 - n_o^2)\sin 2\theta dx dy. \tag{17}$$

With the metric (17), we can find the system of equations

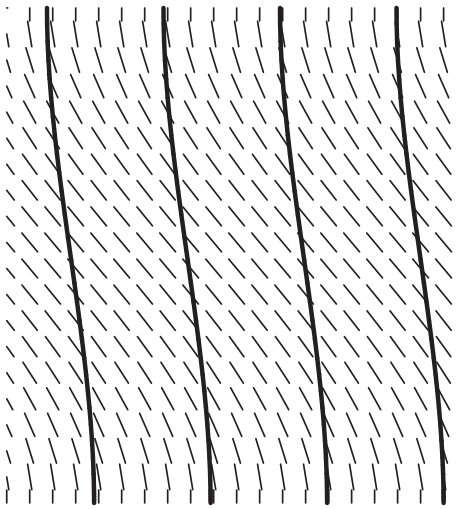

FIG. 2. Light trajectories in the ferronematic cell (solid line) with a field $H=7000$ A/m.

whose solution gives us the light paths in the ferronematic cell through the Eqs. (12) and (13).

IV. DEFLECTION OF THE LIGHT

To solve the geodesics equations, we need the parameter sets that characterize the ferronematic cell built with the 5CB. The refractive indices (10) and (11) of 5CB are characterized by the parameters given in the table below obtained from [12].

A	B	β	$(\Delta n)_0$	T_c
1.7546	0.0005360 K^{-1}	0.2391	0.3768	306.6 K

The parameters A, B, β, and $(\Delta n)_0$ are adimensionals. To define the orientation angle θ of the ferronematic, we have the saturation magnetization $M_s=485$ G, the volume concentration of ferroparticles $f_0=3.6\times10^{-7}$, and the Frank elastic constant $k_{33}=5.3\times10^{-7}$ dyn, whose values are valid for a temperature $T=25$ °C [21]. Under these conditions, we have $n_e=1.701$ and $n_o=1.542$.

The numerical method used in the resolution of the geodesics equations was the Runge-Kutta of the fourth order. In Figs. 2–4, we can see the deflection of light caused by molecular distortion resulting from the application of external field $\vec{H}$. The incidence direction of the beams is the positive direction of the axis $\hat{y}$ in accordance with the coordinates system defined in Fig. 1. In Fig. 2, we have four trajectories (solid line) calculated for an applied field of intensity 7000 A/m and in Figs. 3 and 4, we can see the trajectories obtained previously as the applied field intensity increases to 10000 A/m (dashed line) and 13000 A/m (dotted line), respectively.

The deflection of the beams can be measured in a quantitative form through the scalar curvature [6] perceived by the light. From the metric (17) and of the Christoffel symbols given in Eq. (13), we have that

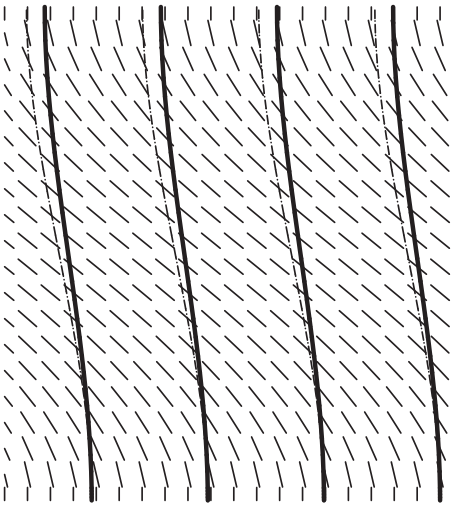

FIG. 3. Light trajectories in the ferronematic cell with $H=10\,000$ A/m (dashed line) along with the previous solution (solid line) to 7000 A/m.

$$R=\xi[Msf_0H(d-2y)^2\cos 2\theta+8k_{33}\sin 2\theta], \tag{18}$$

with

$$\xi=\frac{Msf_0(n_o^2-n_e^2)}{32k_{33}^2n_o^2n_e^2}. \tag{19}$$

Making $n_o=n_e$ in Eq. (18) (isotropic case), we have $R=0$ which is the effective curvature of the flat space. In this case, deflections do not occur.

V. CONCLUSION

In this paper, we have a qualitative prediction of the light behavior in a ferronematic medium through the use of a geometric model constructed from Fermat's principle and differential-geometric tools in Riemannian spaces. The advantage of the geometric method used is in its versatility to

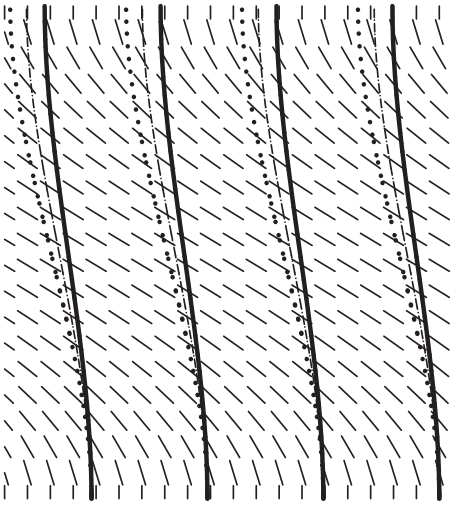

FIG. 4. Light trajectories in the ferronematic cell with a field $H=13\,000$ A/m (dotted line) and the before solutions.

describe the trajectories of light in both ferronematics samples and in nematics with molecular orientation imposed by topological defects [4,22]. Furthermore, the method allows us to find the equations for the trajectories so that the terms $\frac{d^2x^i}{dt^2}$ of Eq. (12) are always given explicitly, unlike other methods (see [23]). The trajectories obtained numerically show the deflections suffered by the light beams traveling across a ferronematic cell with a nonuniform director. The deflection level of the beams is proportional to the intensity of the magnetic field that induces the orientation in the bulk of the cell. As the molecules of the ferronematic have their orientation determined by the anchoring conditions and magnetic field $\vec{H}$ applied, the effect of molecular reorientation due to the interaction between the optical axis and the polarization of the beam can be neglected in case of low-power beams [24,25]. A quantitative evaluation (indirect) of the deflection level of the beams can be inferred through the effective curvature R here calculated. Although the presence of topological defects in nematics causes deviations of the light beams [4], the use of ferronematics as a way to control of the light trajectories is advantageous because the ferronematic samples can easily be produced experimentally in a controlled and stable way [26].

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